

# Two-phase modelling of mushy zone parameters associated with hot tearing

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**Abstract.** A two-phase continuum model for an isotropic mushy zone is presented. The model is based upon the general volume averaged conservation equations, and quantities associated with hot tearing are included; *i.e.*, after-feeding of the liquid melt due to solidification shrinkage is taken into account as well as thermally induced deformation of the solid phase. The model is implemented numerically for a one-dimensional model problem with some similarities to the aluminium direct chill casting process. The variation of some key parameters which are known to influence the hot-tearing tendency is then studied. The results indicate that both liquid pressure drop due to feeding difficulties and shear stress due to tensile deformation caused by thermal contraction of the solid phase are necessary for the formation of hot tears.

## 1 Introduction

Hot tears are a common and serious defect encountered in both ferrous and nonferrous castings. It is generally accepted that hot tears start to develop in the mushy zone at a stage where the solid fraction is close to one. Both Feurer [1] and Rappaz *& al.* [2] stated that hot tearing might result if the pressure in the liquid phase becomes so low that feeding of the total volumetric shrinkage becomes impossible. Pellini [3], on the other hand, stated that hot tearing will result if the material is subjected to a too high accumulated strain within the so-called vulnerable part of the solidification interval, and Clyne and Davies [4] formulated a criterion based upon the time spent in different regimes of the solidification interval. It is referred to Sigworth [5] for a more detailed review on work related to hot tearing.

In accordance with these references, solidification shrinkage leading to interdendritic melt flow is one of the mechanisms associated with hot tearing. The other important mechanism is thermally induced deformation caused by non-uniform cooling contraction of the casting. This motivates for the new two-phase model of an isotropic mushy zone recently formulated by the authors [6]. In this model which is based upon the volume averaged conservation equations [7], both the solid and liquid phases are free to move and interact, and the main focus is on the coherent part of the solidification interval. The purpose of the present paper is to discuss some modelling results obtained by this model.

## 2 Mathematical model

The simplifying assumptions in the new model are thoroughly outlined in ref. [6], and will not be repeated here. Only a few remarks will be made concerning the differences between the

**Tab. 1.** Summary of the mathematical model

<i>Conservation equations</i>			
Mass:	$\frac{\partial(g_s \rho_s)}{\partial t} + \nabla \cdot (g_s \rho_s \mathbf{v}_s) = \Gamma$		
	$\frac{\partial(g_l \rho_l)}{\partial t} + \nabla \cdot (g_l \rho_l \mathbf{v}_l) = -\Gamma$		
Energy:	$(g_s \rho_s C_s + g_l \rho_l C_l) \frac{\partial T}{\partial t} + (g_s \rho_s C_s \mathbf{v}_s + g_l \rho_l C_l \mathbf{v}_l) \cdot \nabla T =$ $\nabla \cdot [(g_s \lambda_s + g_l \lambda_l) \nabla T] + L\Gamma$		
Momentum:	$-g_l \nabla p_l - \mathbf{M} + g_l \rho_l \mathbf{g} = 0$		
	$p_l \nabla g_s - \nabla(g_s p_s) + \nabla \cdot (g_s \boldsymbol{\tau}_s) + \mathbf{M} + g_s \rho_s \mathbf{g} = 0$		
<i>Rheology</i>			
Non-coherent part of the solidification interval:	$p_s = p_l$ $\boldsymbol{\tau}_s = 0$	Coherent mushy zone:	$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s) = 0$ $\frac{\boldsymbol{\epsilon}_s}{\bar{\boldsymbol{\epsilon}}_s} = \frac{3\boldsymbol{\tau}_s}{2\bar{\boldsymbol{\sigma}}_s}$ $\bar{\boldsymbol{\epsilon}}_s = \sqrt{\frac{2}{3} \boldsymbol{\epsilon}_s : \boldsymbol{\epsilon}_s}$ $\bar{\boldsymbol{\sigma}}_s = \sqrt{\frac{3}{2} \boldsymbol{\tau}_s : \boldsymbol{\tau}_s}$ $g_s \bar{\boldsymbol{\sigma}}_s = k(g_s) \bar{\boldsymbol{\epsilon}}_s^{n(g_s)}$
<i>Supplementary relations</i>			
Strain rate:	$\boldsymbol{\epsilon}_s = \frac{1}{2}(\nabla \mathbf{v}_s + [\nabla \mathbf{v}_s]^T) - \frac{1}{3} \mathbf{1} \nabla \cdot \mathbf{v}_s$		
Momentum transfer:	$\mathbf{M} = g_l^2 \mu (\mathbf{v}_l - \mathbf{v}_s) / K(g_l)$		
Lever rule:	$g_l = \frac{\rho_s (c_0 - k c_l(T))}{c_0(\rho_s - \rho_l) - c_l(T)(\rho_l - k \rho_s)}, \quad c_l(T) = (T - T_m) / m$		
Permeability:	$K = K_0 g_l^3 / (1 - g_l)^2$		
<i>Nomenclature</i>			
$g$ volume fraction	$\Gamma$ interfacial mass transfer	$m$ slope of liquidus line	
$\mathbf{v}$ velocity	$\mathbf{M}$ interfacial momentum transfer	$k$ partition coefficient	
$T$ temperature	$K$ permeability	$k(g_l)$ creep law parameter	
$p$ pressure	$K_0$ permeability constant	$n(g_l)$ creep law parameter	
$\boldsymbol{\tau}$ shear stress	$\rho$ mass density		
$\bar{\boldsymbol{\sigma}}$ effective stress	$C$ heat capacity		
$\boldsymbol{\epsilon}$ strain rate	$\lambda$ heat conductivity	index $s$ solid	
$\bar{\boldsymbol{\epsilon}}$ effective strain rate	$L$ latent heat	index $l$ liquid	
$\mu$ viscosity	$\mathbf{g}$ gravity	index $mp$ melting point	

present model and two-phase models applied for other fluid flow phenomena in the mushy zone. The equations of the model are summarised in Tab. 1.

While the liquid density is considered constant (free convection is beyond the scope of the model), the solid density is taken to be a linear function of the temperature in order to introduce the thermal contraction. For thermally induced deformations in the solid, momentum transfer due to acceleration is negligible. This is clearly also the case for the momentum transfer in the liquid when the mushy zone is coherent. Furthermore at small liquid fractions, diffusion of momentum in the liquid phase is negligible compared to the momentum transfer due to

dissipative interfacial forces. This simplifies the momentum equation for the liquid to yield Darcy’s law.

While the interfacial liquid pressure can be set equal to the bulk liquid pressure (due to instantaneous pressure equilibration), a similar simplification cannot be made for the solid phase. This is because an additional pressure can be transmitted through the coherent solid structure. Above the coherency temperature the solidified grains are assumed to move freely in the liquid, and it can be assumed that the pressure is equal in the two phases [7]. Mechanically, this means that the solid structure poses no restriction against isotropic compression/densification. It should be noted that this does not necessarily mean that the velocities of the two phases are the same, since the solidified grains can settle due to differences in density. In the coherent part of the solidification interval, this assumption is not valid since an additional pressure can be transmitted through the solid phase. In the present work it is assumed that the coherent network is incompressible and connected in a manner such that the thermal contractions of the solid phase must be compensated for solely by deformations of the solid structure. Mathematically this means that the solid phase must obey the single-phase continuity equation.

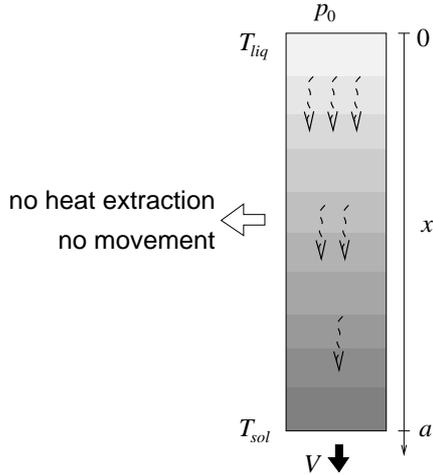
In the (coherent) mushy zone, the thermally induced deformations (which are assumed to take place in the solid phase only) are taken to be inelastic. The volume averaged deviatoric viscoplastic strain rate in the solid phase is then related to the deviatoric stress tensor by the Levy–Mises flow law, since the material is assumed to be isotropic. Several authors have measured the rheological behaviour in partially solidified aluminium alloys, see, *e.g.*, refs. [8–10]. From these works, it seems reasonable to relate the effective stress of the solid phase to the effective strain rate by a creep law. In the present work, the creep law is chosen as a simple power law.

### 3 One-dimensional test problem

Consider the one-dimensional stationary Bridgman-like casting process shown in Fig. 1. At the bottom where the material is entirely solidified, solid material is taken out at a constant casting speed. At this position, the solidus temperature is imposed as a boundary condition. Melt with a temperature equal to the liquidus temperature flows into the domain at the top. Due to solidification shrinkage and cooling contraction of the solid phase, the vertical liquid velocity at the top is slightly higher than the casting speed. It is assumed that all transport phenomena occur in one direction only, *viz.* along the axis of solidification, and that the gravity can be neglected. This means, in addition to no heat extraction in the horizontal direction, that the contracting material is restricted from contracting horizontally. Thus, stress will arise, trying to tear the material apart along the axis.

It should be noted that this simple stationary one-dimensional test problem has several analogies to the situation in the centre of a direct chill casting process where the mushy zone is restricted to move in the vertical direction due to the presence of a solidified shell surrounding the solidifying region. Furthermore, if the sump is not too deep and curved in the centre, heat extraction mainly occurs along the axis.

The one-dimensional equations have been solved for an Al4.5%Cu alloy under conditions relevant for the direct chill casting process. The parameters given as input to the model for this case are listed in Tab. 2. The resulting stress and pressure in the two phases are shown in Fig. 2. The upper curve shows the effective stress in the solid phase which decreases rapidly from its value at the solidus to zero at coherency. The absolute values of the pressures in the solid and liquid phases show a similar behaviour. In the region in the mushy zone where hot tears might form, *i.e.*, at liquid fractions between 0.01 and 0.1, the liquid pressure is lower than the solid pressure. It is therefore reasonable to argue that hot tears do not form as a consequence



**Fig. 1.** One-dimensional test problem.

**Tab. 2.** Input parameters for the default case

$V = 10^{-3}$ m/s	Casting speed
$a = 10^{-2}$ m	Length of mushy zone
$M = 5000$	Number of nodes
$p_0 = 0$	Metallostatic pressure
$g_{l,coh} = 0.5$	Liquid fraction at coherency
$\beta_s = 0.0658$	Solidification shrinkage
$\beta_T = -9 \cdot 10^{-5}$ K <sup>-1</sup>	Thermal expansion
$c_0 = 0.045$	Concentration of Cu
$c_e = 0.33$	Concentration at eutectic
$k = 0.17$	Partition coefficient
$m = -339$ K	Slope of liquidus line
$T_{mp} = 933$ K	Melting point (pure Al)
$T_e = 821$ K	Eutectic temperature
$C_l = 1060$ J/(kg K)	Specific heat in liquid
$C_s = 1060$ J/(kg K)	Specific heat in solid
$\lambda_l = 83$ W/(m K)	Heat conductivity in liquid
$\lambda_s = 192$ W/(m K)	Heat conductivity in solid
$L = 4 \cdot 10^5$ J/kg	Latent heat

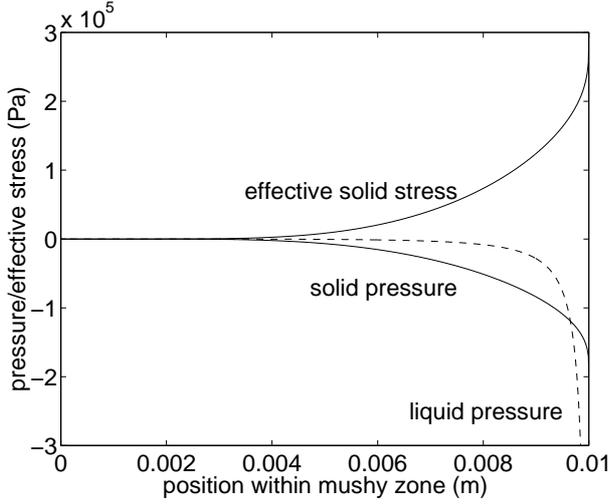
of hydrostatic depression only (although pore formation might be the result of the low liquid pressure). Instead, tensile stress is required, as pointed out by Campbell [11].

The hot-tearing susceptibility is known to depend critically upon the solidification interval [2,4], the thermal contraction of the solid phase [3], the liquid fraction at coherency [12], and, in the case of direct chill casting, the casting speed. Case studies in which these four parameters are varied have therefore been performed. Since variations in these parameters affect the hot tearing tendency, they should result in variations in key parameters, *e.g.*, stress and pressure, in the present model. Following Clyne and Davies [4], the values of the stress and pressure at a “critical point” in the mushy zone where the liquid fraction equals 1% will be examined.

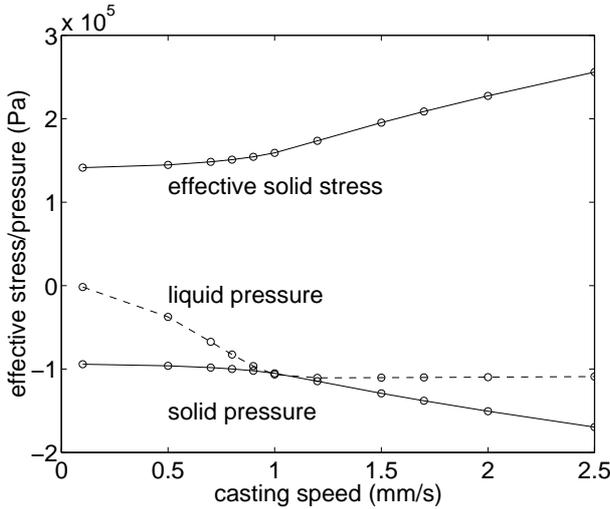
Variations in the composition of the alloy result in variations in the solidification interval. Fig. 3 shows the effect of varying the amount of copper in the binary Al–Cu alloy on the liquid pressure at the critical point under otherwise identical casting conditions. The so-called lambda curve (see, *e.g.*, refs. [2, 4, 11]) is reproduced, indicating a peak in the pressure for a certain alloy composition at which hot tearing is most likely to occur. The effective stress and pressure in the solid phase is, on the other hand, not affected by the variations in composition since it is mainly a function of the cooling rate. This indicates that a sufficient drop in the liquid pressure is necessary for the formation of hot tears.

When the same numerical experiment is performed on an artificial alloy which is similar to the Al–Cu system in all respects except that there is no cooling contraction of the solid phase, nearly the same result is obtained in terms of the liquid pressure (dashed line in Fig. 3). In this alloy, there is obviously no stress or strain in the solid phase whatsoever. One would therefore not expect hot tearing, but instead porosity formation [11]. A hot-tearing criterion based upon the liquid pressure would, on the other hand, predict almost the same hot-tearing tendency for the two cases. This indicates that the liquid pressure drop cannot constitute the full basis for a hot-tearing criterion.

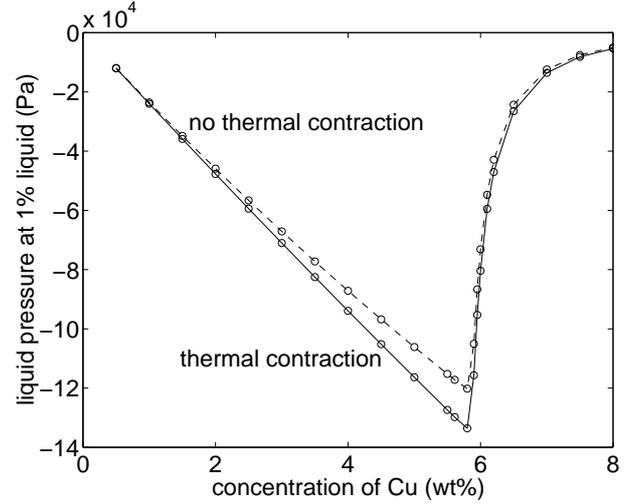
When varying the casting speed, the effective stress in the solid phase and the pressure in both phases at the critical point varies as shown in Fig. 4. It is observed that the liquid pressure at the critical point decreases rapidly with increasing casting speed until a certain point where the effect suddenly stops. This is when the terms for convection and release of latent heat become dominating in the energy equation. It is also observed that the negative solid pressure



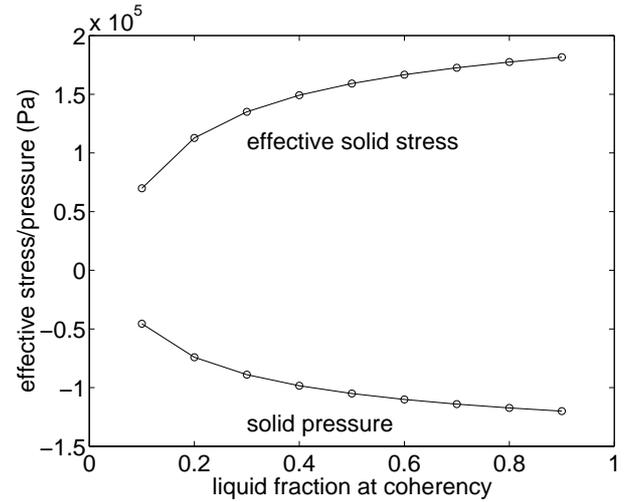
**Fig. 2.** Pressure and stress in the two phases resulting from running the model on the default case plotted as a functions of the position within the mushy zone.



**Fig. 4.** The effect of the casting speed on effective stress in the solid phase and on the pressure in both phases at the critical point within the mushy zone.



**Fig. 3.** Liquid pressure drop as a function of composition in a binary Al-Cu alloy for otherwise identical casting conditions (solid line), and for the same system with no cooling contraction of the solid phase (dashed line).



**Fig. 5.** The effect of the liquid fraction at coherency on pressure and effective stress in the solid phase at the critical point within the mushy zone.

and the effective stress of the solid are increasing with increasing speed.

An input parameter to the model is the value of the liquid fraction at coherency. According to experiments, an increase in this value leads to an increased hot-tearing susceptibility. However, the model shows that the liquid pressure at the critical point is completely unaffected by this variation in the liquid fraction at coherency. This is because the negative liquid pressure builds up very near the end of solidification, and is almost unaffected of whether it starts building up at an early or late coherency. Thus, a hot-tearing criterion formulated in terms of the liquid pressure alone will not reflect the effect of a variation in the liquid fraction at coherency.

On the other hand, Fig. 5 shows that the liquid fraction at coherency has a strong impact upon the stress and pressure in the solid phase. This is because coherency at a high liquid fraction gives rise to a large coherent solidification range in which stress can build up in the solid phase. It is therefore reasonable to argue that a theory for hot tearing must take the behaviour of the solid phase into account along with the liquid pressure.

## 4 Conclusions

A two-phase continuum model for an isotropic mushy zone has been developed. In this model, after-feeding of the liquid melt due to solidification shrinkage is taken into account as well as thermally induced deformation of the solid phase. Results from a one-dimensional model problem reveals:

- The pressure in the solid phase is higher than the pressure in the liquid phase close to the end of solidification. This indicates that tensile stress is necessary for the formation of hot tears.
- The so-called lambda curve is reproduced for the liquid pressure *versus* alloy composition. This indicates that feeding difficulties is important for the formation of hot tears.
- The liquid pressure is more or less unchanged for an artificial alloy with no cooling contraction in the solid phase. This indicates that a liquid pressure drop above some critical value cannot be the only parameter in a hot-tearing criterion.
- Increasing the casting speed has a great impact on stress and pressure in both the solid and liquid phases.
- A variation in the liquid fraction at coherency does not affect the liquid pressure, whereas it is of major importance for the stress and pressure in the solid phase.

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