Riemannian Formulation of the CIEDE2000 Color Difference Formula

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Abstract

The CIELAB based CIEDE2000 colour difference formula to measure small to medium colour differences is the latest standard formula of today which incorporates different corrections for the non uniformity of CIELAB space. It also takes account of parametric factors. In this paper, we present a mathematical formulation of the CIEDE2000 by the line element to derive a Riemannian metric tensor in a color space. The coefficients of this metric give Just Noticeable Difference (JND) ellipsoids in three dimensions and ellipses in two dimensions. We also show how this metric can be transformed between various colour spaces by means of the Jacobian matrix. Finally, the CIEDE2000 JND ellipses are plotted into the xy chromaticity diagram and compared to the observed BFD-P colour matching ellipses by a comparing method described in Pant and Farup (CGIV2010).

Introduction

A color difference formula or a color difference metric, which measures the difference between two colors, is becoming a prime research topic in the modern colorimetry. The target is to find a good color difference formula, which can give a quantitative measure (ΔE) of the perceived color difference correctly. It is also the requirement of many color applied fields such as image analysis, color reproduction, color image restoration and so on.

MacAdam [2] described that small or medium color differences can be measured by the Just Noticeable Difference (JND) ellipses, which represent the human perception of threshold colour differences. Since then, many color difference formulae have been developed for measuring the color difference accurately. In 1976, the CIE recommended two color difference formulae, the CIELAB and CIELUV formulae, have become popular in colour industries, but, they fail to measure the visual perception of the color differences sufficiently, although, they are said to be uniform colour difference formulae [3, 4].

In 2001, the CIE recommended the CIEDE2000 formula based on the CIELAB to improve the correlation between measured and human observed color differences. In particular, the CIEDE2000 is the improved version of the CIELAB with specific weighting functions known as lightness (S_L), chroma (S_C) and hue (S_H), parametric factors (k_L, k_C, k_H), and the rotation term R_T to correct chroma and hue differences in the blue region. All these modifications are based on visual data obtained from four different experiments known as BFD-P [5], Leeds [6] RIT-DuPont [7] and Witt [8]. In other words, visual results from these four experiments were adjusted to a common scale by computing scaling factors for each data set and adopting the visual scale of BFD-P as a unit [9]. Luo et al. [10] have described in the detail about the CIEDE2000 formula as an excellent out performing formula, when measured against the aggregate data set, but still it has some issues related to its development [11]. Similarly, Sharma et al. [12] have shown mathematical discontinuity in the formula. Further, field test reports and performance studies on the CIEDE2000 have also not shown conclusively that the latest CIE formula performs better than previous existing formulae [13–15].

In such contexts, it would be useful from many aspects to study the CIEDE2000 color difference formula by the Riemannian approach. First, in this approach, we can map or transfer this formula into other color spaces preserving the subjective property of the formula. Second, the formula does not have its specific or corresponding color space, it is only the improved L, a^*, b^* color space formulated in terms of lightness (L), chroma (C) and hue (H). So, it is interesting to know how well this advanced formula measures small colour differences in other color spaces. Third, Riemannian space is curved and such space is considered suitable for small to medium color difference measurement because many researchers have described that small color difference calculation using the Euclidean distance does not agree sufficiently with the perceptual color difference due to the curvilinear nature of the color space [16–20].

In this paper, the authors present a method to formulate the CIEDE2000 color difference formula in terms of the Riemannian metric and this metric is used to compute the JND ellipses. Here, the authors take the line element to calculate the color differences dE. To calculate line element, the CIEDE2000 color difference equation should be converted into the differential form. This gives us the Riemannian metric in a non-Euclidean color space. Again, to obtain the Riemannian metric in a new color space, we also need to transform color vectors from one color space to another. This is accomplished by the Jacobian transformation. To illustrate our method, the authors transformed the CIEDE2000 formula into the xyY color space. And, the JND ellipses are plotted into the xy chromaticity diagram. The input data to compute the JND ellipses for our method is BFD-P data sets [5]. BFD-P data sets were assessed by about 20 observers using a ratio method, and the chromaticity discrimination ellipses were calculated and plotted in the xy chromaticity diagram for each set [21]. A comparison has also been done between the computed JND ellipses of the CIEDE2000 formula and the original ellipses obtained from the BFD-P data set. The detailed description, for comparing a pair of ellipses, by calculating the ratio of the area of the intersection and the area of the union can be found in [1]. This method gives a single comparison value which takes account of variations in the size, the shape and the orientation simultaneously for a pair of ellipses. Therefore, this value is an indicator which tells us how well two ellipses match each other. Further, using MacAdam data, the authors also plot the JND ellipses of the formula in the xy chromaticity diagram to see simple visual comparison with the original data set. The authors see a good mathematical technique in this method to study the CIEDE2000 color difference formula.

Formulation of the CIEDE2000 Metric Tensor and Color Space Transformation

In this section, the authors will describe our method to compute metric tensor of the CIEDE2000 formula in the xyY space. Let us begin the process by defining the standard form of the formula [22].

$$\Delta E_{00} = \left[\left(\frac{\Delta L'}{k_L S_L} \right)^2 + \left(\frac{\Delta C'}{k_C S_C} \right)^2 + \left(\frac{\Delta H'}{k_H S_H} \right)^2 + R_T \left(\frac{\Delta C'}{k_C S_C} \right) \cdot \left(\frac{\Delta H'}{k_H S_H} \right) \right]^{0.5} \quad (1)$$

In Equation (1), R_T is the rotation function and expressed as $R_T = -\sin(2\Delta\theta) R_c$

where $\Delta\theta = 30 \cdot exp - \left(\frac{\bar{h}' - 275}{25} \right)^2$ and $R_c = 2\sqrt{\frac{\bar{C'}^7}{\bar{C'}^7 + 25^7}}$

Similarly, the weighting functions are defined as:

$$S_L = 1 + \frac{0.015(\bar{L}' - 50)^2}{\sqrt{20 + (\bar{L}' - 50)^2}}$$

$S_C = 1 + 0.045\bar{C}'$ and $S_H = 1 + 0.015\bar{C}'T$ with

$T = 1 - 0.17\cos(\bar{h}' - 30^o) + 0.24\cos(2\bar{h}' + .32\cos(3\bar{h}' + 6^o) - 0.2\cos(4\bar{h}' - 63^o)$

Here, we define for a pair of color samples $\bar{L}' = \frac{L_1^* + L_2^*}{2}$

$\bar{C}' = \frac{C_1' + C_2'}{2}$ and $\bar{h}' = \frac{h_1' + h_2'}{2}$ and $\Delta H = 2\sqrt{C_1' C_2'} \sin \frac{\Delta h'}{2}$

The other symbols used in the formula are also defined in the following way:

$L' = L^*,\ a' = a^*(1 + G),\ b' = b^*$ and $C' = \sqrt{a'^2 + b'^2}$ with

$h' = \arctan \frac{b'}{a'}$ and $G = \frac{1}{2}\left(1 - \sqrt{\frac{C*_{ab}^7}{C*_{ab}^7 + 25^7}} \right)$

where L^*, a^* and b^* corresponds to the Lightness, the redness-greenness and the yellowness-blueness scales and C^* chroma in the CIELAB color space. Likewise, h' is the hue angle for a pair of samples. To formulate Riemannian Metric, the authors take only L', C' and h' values rather than their arithmetic mean values $\bar{L}'$, $\bar{C}'$ and $\bar{h}'$ because in the Riemannian or non-Euclidean color space, infinitesimal distance is taken to measure colour differences. So, Equation (1) becomes a differential form as follows:

$$(dE_{00}^*)^2 = \begin{bmatrix} dL' & dC' & dH' \end{bmatrix} \times \begin{bmatrix} (k_L S_L)^{-2} & 0 & 0 \\ 0 & (k_C S_C)^{-2} & \frac{1}{2}(k_C S_C k_H S_H)^{-1} \\ 0 & \frac{1}{2}(k_C S_C k_H S_H)^{-1} & (k_H S_H)^{-2} \end{bmatrix} \times \begin{bmatrix} dL' \\ dC' \\ dH' \end{bmatrix} \quad (2)$$

In Equation (2), the matrix of coefficients of weighting functions, parametric factors, and the rotation term is the Riemannian metric of the formula in its original form. As said in the introduction section, the authors will show the process to transform it into the xyY space in terms of metric form. In general, the transformation process takes the following steps: First, from $L'C'H'$ to $L^*a^*b^*$ then to XYZ tristimulus color space and finally into the xyY color space.

So, at first, we need to transform differential color vectors dL, dC', dH' into dL^*, da^*, db^*. Since, they are different color vectors or functions at a given point in a color space, we can only relate them by applying the Jacobian method. In this method, we compute all partial derivatives of vector functions L', C', and H' with respect to L^*, a^*, and b^*. This gives us a 3×3 matrix of continuous partial derivatives, which is known as a Jacobian matrix.

In the equation form, we write:

$$\begin{bmatrix} dL' \\ dC' \\ dH' \end{bmatrix} = \begin{bmatrix} \frac{\partial L'}{\partial L^*} & \frac{\partial L'}{\partial a^*} & \frac{\partial L'}{\partial b^*} \\ \frac{\partial C'}{\partial L^*} & \frac{\partial C'}{\partial a^*} & \frac{\partial C'}{\partial b^*} \\ \frac{\partial H'}{\partial L^*} & \frac{\partial H'}{\partial a^*} & \frac{\partial H'}{\partial b^*} \end{bmatrix} \begin{bmatrix} dL^* \\ da^* \\ db^* \end{bmatrix} \quad (3)$$

where, the matrix of partial derivatives in Equation (3) can also be denoted by $\frac{\partial(L', C', H')}{\partial(L^*, a^*, b^*)}$. Now, the Equation (2) becomes as follows:

$$(dE_{00}^*)^2 = \begin{bmatrix} dL^* & da^* & db^* \end{bmatrix} \frac{\partial(L', C', H')}{\partial(L^*, a^*, b^*)}^T \times \begin{bmatrix} (k_L S_L)^{-2} & 0 & 0 \\ 0 & (k_C S_C)^{-2} & \frac{1}{2}(k_C S_C k_H S_H)^{-1} \\ 0 & \frac{1}{2}(k_C S_C k_H S_H)^{-1} & (k_H S_H)^{-2} \end{bmatrix} \times \frac{\partial(L', C', H')}{\partial(L^*, a^*, b^*)} \begin{bmatrix} dL^* \\ da^* \\ db^* \end{bmatrix} \quad (4)$$

The calculation of $\frac{\partial(L', C', H')}{\partial(L^*, a^*, b^*)}$ appears as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\partial C'}{\partial a^*} & \frac{\partial C'}{\partial b^*} \\ 0 & \frac{\partial H'}{\partial a^*} & \frac{\partial H'}{\partial b^*} \end{bmatrix} \quad (5)$$

where $dH' = C'dh'$ because $\lim\ \Delta H \to 0 \sin \frac{\Delta h'}{2} \approx \frac{\Delta h'}{2} \Rightarrow dH' =$

$2C'dh/2'$

$$
\begin{aligned}
\frac{\partial C'}{\partial a^*} &= \frac{a'}{C'}\left[(1+G)+\frac{a^{*2}}{C^*}\left(-\frac{175\times 25^6}{4}\frac{C^{*5/2}}{(C^{*7}+25^7)^{3/2}}\right)\right]\\
\frac{\partial C'}{\partial b^*} &= \frac{b'}{C'}+\frac{a'}{C'}\left[\frac{a^*b^*}{C^*}\left(-\frac{175\times 25^6}{4}\frac{C^*5/2}{(C^{*7}+25^7)^{3/2}}\right)\right]\\
\frac{\partial H'}{\partial a^*} &= C'\frac{\partial h'}{\partial a^*}-\frac{b'}{C'}\left[1+G+\frac{a^{*2}}{C^*}\left(-\frac{175\times 25^6}{4}\frac{(C^*)^{5/2}}{((C^*)^7+25^7)^{3/2}}\right)\right]\\
\frac{\partial H'}{\partial b^*} &= C'\frac{\partial h'}{\partial b^*}=\frac{a'}{C'}
\end{aligned}
\tag{6}
$$

In a similar way, $\frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}$ Jacobian matrix map color vectors L^*, a^*, b^* into X, Y, Z tristimulus color space and by another Jacobian matrix $\frac{\partial(X,Y,Z)}{\partial(x,y,Y)}$, we can relate X, Y and Z tristimulus and x, y and Y color vectors. The detailed derivations of these Jacobians can be found in [1]. Finally, the mapping of the CIEDE2000 formula into the xyY color space in terms of the metric tensor is

$$
\begin{aligned}
(dE_{00})^2 = \begin{bmatrix} dx & dy & dY \end{bmatrix} & \frac{\partial(X,Y,Z)}{\partial(x,y,Y)}^T \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}^T \frac{\partial(L',C',H')}{\partial(L^*,a^*,b^*)}^T \times \\
& \begin{bmatrix} (k_LS_L)^{-2} & 0 & 0 \\ 0 & (k_CS_C)^{-2} & \frac{1}{2}(k_CS_Ck_HS_H)^{-1} \\ 0 & \frac{1}{2}(k_CS_Ck_HS_H)^{-1} & (k_HS_H)^{-2} \end{bmatrix} \times \\
& \frac{\partial(L',C',H')}{\partial(L^*,a^*,b^*)} \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)} \frac{\partial(X,Y,Z)}{\partial(x,y,Y)} \begin{bmatrix} dx \\ dy \\ dY \end{bmatrix}
\end{aligned}
\tag{7}
$$

Multiplying all the Jacobian matrices, their transposes and the matrix of original form (matrix of correction terms) together, we will get a 3×3 matrix in the xyY color space.

This matrix is known as the Riemannian metric tensor (g_{ik}) of the CIEDE2000 formula in the xyY color space, which gives JND ellipsoids in three dimensions and ellipses in two dimensions. The principal axes of ellipses can be calculated from eigenvectors and eigenvalues of the metric g_{ik}. So, if λ_1 and λ_2 are eigenvalues of the g_{ik}, the axis (a) and the axis (b) equal to $\frac{1}{\sqrt{\lambda_1}}$ and $\frac{1}{\sqrt{\lambda_2}}$ respectively.

Result and Discussion

In this section, the authors will discuss on the behavior of computed ellipses of the formula in the xyY color space with respect to the BFD-P ellipses.

One severe problem is found at the gray axis where the Jacobian (Equation 5) is not defined. It can be easily seen by inserting $a*$ and $b* = 0$ in Equations (6. In fact, the limit does not exist, since it depends on the path. Thus, a Riemannian metric does not exist at the gray axis. However, JND ellipses can be computed by the metric defined in Equation (7) for the rest of the colour space.

Here, all the calculated or computed ellipses of the CIEDE2000 formula and BFD-P ellipses have same color centers. Again, to draw the computed ellipses into the xy chromaticity diagram, the authors have taken the constant lightness value

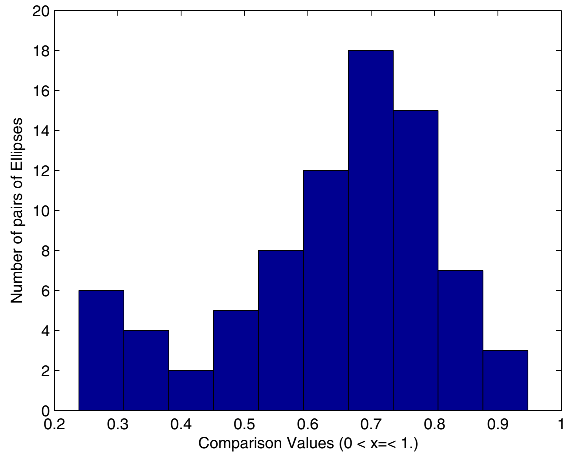


Figure 1: Histogram of comparison values between CIEDE2000 and BFD-P ellipses. The values lie in the range $0 < x \leq 1$. Higher comparison value indicates better matching between a pair of ellipses.

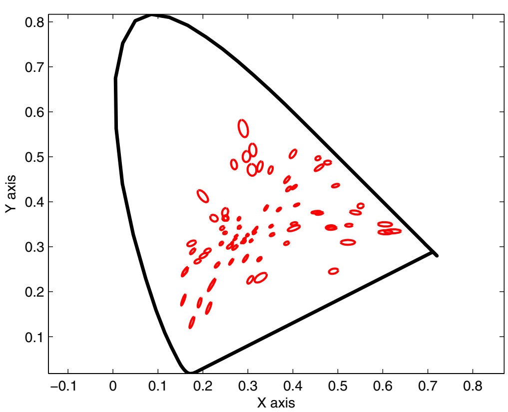
($L^* = 50$) and this value equals to the lightness value which was taken to draw original BFD-P ellipses. The variables (k_L, k_C and k_H) are set to 1 for calculating ellipses. Now, at first, the authors describe qualitative analysis between computed and original ellipses. Figures 2(a) and 2(b) show BFD-P and computed CIEDE2000 ellipses drawn in the CIE64 chromaticity diagram respectively. It can be seen that ellipses for the neutral and gray color centers are almost the same in both figures. Similarly, in both BFD-P and CIEDE2000, the ellipses for blue, green-blue, green-yellow, yellow and red centers tend to point along lines of constant dominant wavelength. However, in CIEDE2000, the orientation of ellipses in the blue region are rotated compared to the BFD-P ellipses of same region. On the other hand, in the red region too, the CIEDE2000 ellipses are rotated in opposite direction and stretched in length. In terms of size, the CIEDE2000 ellipses in blue and green-blue are slightly smaller than the BFD-P ellipses in the corresponding region, where as they are slightly larger as well as more circular in shape in the yellow region.

Our next analysis between CIEDE2000 and BFD-P is based on our method for comparing the similarity of a pair of ellipses as said in the introduction section. The value obtained by this method lies in the range of $0 < x \leq 1$. Hence, a comparison value of 1 between a pair of ellipses ensures the full compatibility between them in terms of size, shape and orientation. According to this method, the authors have got maximum matching value of .95 between a pair of CIEDE2000 and BFD-P ellipses. This pair appears in the neutral color region. Similarly, the minimum value has come .25 around high chroma blue. The matching values of all ellipse pairs can be seen in the histogram 1 as well as from the Table 1.

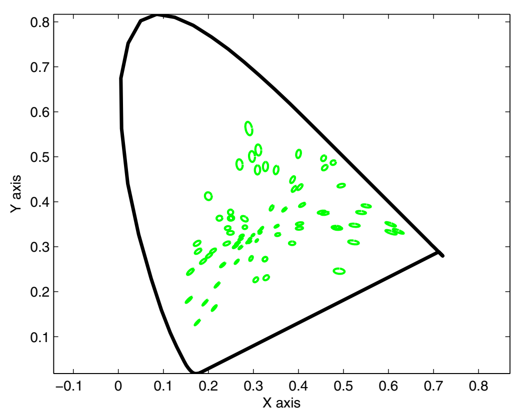
Similarly, Figures 3(c) and 3(b) show the MacAdam's original ellipses and the CIEDE2000 ellipses taking his original color centers in the 1931 xy chromaticity diagram. Ellipses are plotted at constant lightness level at $L^* = 50$. These figures also help to visualize the difference between original and computed ellipses in terms of size shape and orientation in a simple manner and the general trend of difference is also similar to the BFD-P and the CIEDE2000 ellipses as described above.

Table 1: Comparison values of CIEDE2000 and BFD-P ellipses pairs . A comparison value of 1 between a pair of ellipses ensures the full compatibility between them in terms of size, shape and orientation.

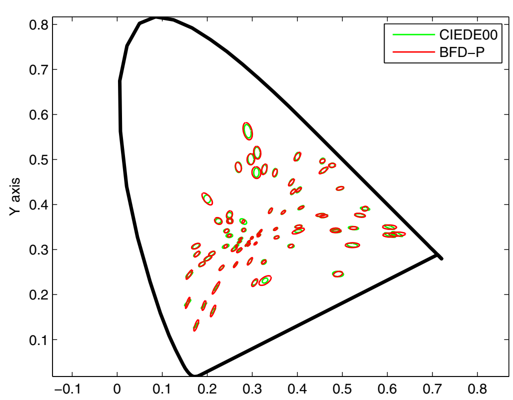
Ellipse pair number	1	2	3	4	5	6	7	8	9	10	11	12
Value	0.96	0.93	0.9	0.88	0.88	0.88	0.86	0.85	0.85	0.83	0.82	0.81
Ellipse pair number	13	14	15	16	17	18	19	20	21	22	23	24
Value	0.79	0.79	0.78	0.77	0.77	0.77	0.76	0.75	0.75	0.75	0.75	0.75
Ellipse pair number	25	26	27	28	29	30	31	32	33	34	35	36
Value	0.75	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.72	0.72	0.71	0.71
Ellipse pair number	37	38	39	40	41	42	43	44	45	46	47	48
Value	0.71	0.71	0.7	0.7	0.69	0.69	0.69	0.69	0.68	0.68	0.68	0.68
Ellipse pair number	49	50	51	52	53	54	55	56	57	58	59	60
Value	0.66	0.66	0.65	0.65	0.64	0.62	0.6	0.58	0.58	0.57	0.54	0.53
Ellipse pair number	61	62	63	64	65	66	67	68	69	70	71	72
Value	0.53	0.53	0.52	0.52	0.51	0.5	0.49	0.48	0.47	0.46	0.46	0.44
Ellipse pair number	73	74	75	76	77	78	79	80				
Value	0.4	0.38	0.37	0.34	0.34	0.33	0.33	0.29				



(a) BFD-P ellipses.

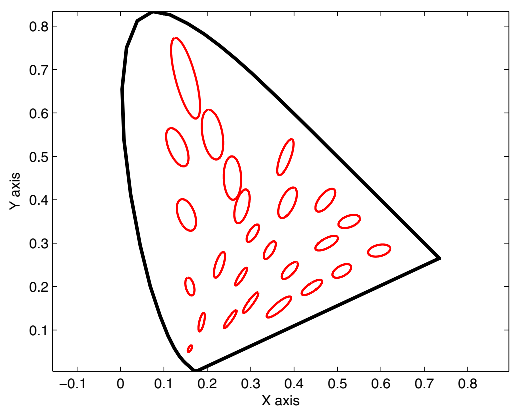


(b) CIEDE2000 ellipses having same color centers as BFD-P.

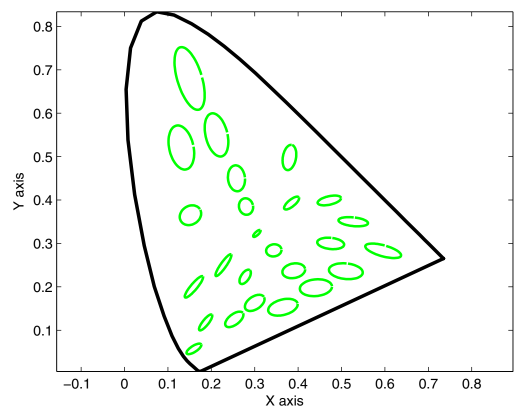


(c) BFD-P and CIEDE2000 ellipses plotted on the same xy diagram .

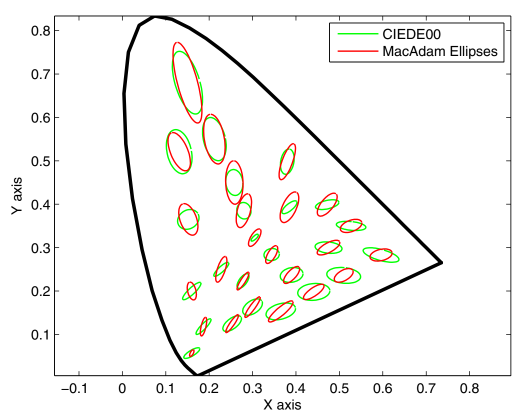
Figure 2: BFD-P ellipses and Computed CIEDE2000 Ellipses in the CIE64 Chromaticity diagram (Enlarged 1.5 times)



(a) MacAdam Ellipses.



(b) CIEDE2000 Ellipses having same color centers as MacAdam.



(c) MacAdam and CIEDE2000 Ellipses plotted on the same xy diagram.

Figure 3: MacAdam's original and Computed CIEDE2000 ellipses in the CIE31 Chromaticity diagram (Enlarged 10 times)

Conclusion

The first objective of this paper is to formulate the CIEDE2000 formula into the Riemannian metric and apply the Jacobian method to transfer it into different color spaces as well as to compute JND ellipses from this metric. This is successfully accomplished, except at the gray axis. Second objective is to study the behavior of the formula in the xyY color space with respect to the experimentally observed data. This is also done by drawing JND ellipses of the formula and experimentally observed BFD-P ellipses into the xy chromaticity diagram and comparing them by our comparison technique described above.

On the basis of our findings as discussed above, the authors can say that the CIEDE2000 significantly measures the visual color differences. However, it is seen orientation problem in the CIEDE2000 ellipses compared to BFD-P ellipses in the blue region as well as in the red region. This indicates that further research for the improvement of the rotation term or the colour difference metrics, in general is necessary. Our method has also shown that the formula measures small color difference well in the non-Euclidean space.

The authors hope that the formula presented here will be useful for the color research and applications.

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