CIE UNIFORM CHROMATICITY SCALE DIAGRAM FOR MEASURING PERFORMANCE OF OSA-UCS ΔEE AND CIEDE00 FORMULAS

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ABSTRACT

The CIE recommended uniform chromaticity scale (UCS) diagram based on the CIELUV is used to evaluate the non-Euclidean approximate form of CIEDE2000 and the Euclidean ΔE_E colour difference formulas for measuring the visual data. Experimentally observed visual colour difference data in terms of supra threshold ellipses are plotted in the CIELUV u^*, v^* diagram. Similarly, equi-distance ellipses of two formulas are computed and plotted in the same diagram. Performance of these formulas are evaluated by calculating the matching ratio between observed and computed ellipse pairs. Various statistical tests are done for these ratio values. It is found that there is no significant difference between the complex non-Euclidean approximate form of ΔE_{00} and the simple Euclidean ΔE_E.

***Index Terms*—** CIEDE2000, OSA-UCS ΔE_E, ellipses, visual colour difference

1. INTRODUCTION

Among the many colour difference formulas so far developed, the CIEDE2000 [1] and ΔE_E proposed by Oleari [2] based on the OSA-UCS color space are considered more robust and latest. The first one is an improved formula based on the CIELAB color space, resulting in a non-Euclidean metric. The second is the recent Euclidean color difference metric defined in OSA-UCS space with interger values of lightness L in a regular rhombohedron planes [2]. These two color difference formulas are established with different criteria. It is interesting to know how the two color difference formulas perform for measuring the visual data.

The visual data is the experimental color discrimination data derived from visual color difference experiments. They are described by supra threshold ellipses. In general, these data represent the sum of a number of threshold differences along a particular path between two colors called line element [3]. MacAdam [4] was the first to study about discrimination data and from this data he constructed just noticeable difference (JND) ellipses. Later, similar data sets were derived by Brown [5], Wyszecki and Fielder [6]. Similarly, more advanced data sets such as BFD-Perceptibility(BFD-P) [7] were developed based on supra threshold color differences. Bigg and Luo established BFD-P as a standard data set for surface colors represented in the xyY colour space [7]. On the other hand, if we treat a color space as a Riemannian space [8], the ellipses of the approximate form of CIEDE2000 [1] and ΔE_E can be computed in any other colour space, as shown by Pant et. al. [9]. These computed ellipses can be compared with experimentally obtained ellipses. However, comparing ellipses in xy chromaticity scale diagram have limitations due to the large deviation from uniform spacing [10–12].

To compare visual differences, a uniform color space is very important because in such a space a unit change in hue, chroma and lightness are equal. In practice, realization of these scales are difficult. As a result, many uniform color spaces are proposed such as CIELUV [13], ATD [14] OSA 90 [15] and so on. The CIE recommended CIELUV based uniform chromaticity scale (UCS) diagram is developed by projective transform of the CIE tristimulus (XYZ) values. Because of this, the CIELUV chromaticity u^*, v^* follows the property of linearity. Further, the aim of CIELUV space is to map MacAdam's JND ellipses as close to circles, it is more suitable for measuring small colour differences. Since these formulas are optimized to predict visual color differences in their specific colour spaces, it is also desirable to know how well they perform to accomplish such tasks in an independent uniform colour space like CIELUV. In the literature, many other researchers also suggest to use u^*, v^* diagram before drawing any conclusions about visual differences [16–18].

In this paper, the authors compute equi-distance ellipses of the CIEDE2000 and ΔE_E in the u^*, v^* diagram. The CIEDE2000 is treated here as the Riemannian approximation considering L, C, h coordinates instead of the formula defined L, C, H. The detailed explanation about this can be found in the authors previous article [19]. They are compared with the experimental BFD-P data which is also plotted in the CIELUV chromaticity u^*, v^* diagram. We take an infinitesimal distance to compute the ellipses of approximate CIEDE2000 and ΔE_E formulas. In this process, color difference equations are converted into the differential form. This gives metric tensors (g_{ik}) of above mentioned two formulas. Ellipse parameters, major axes and angle of rotation can be determined by this metric tensor and vice versa. The Jacobian transformation method is used to transform computed metric from one colour space to another. For example, The BFD-P data is plotted in the xy chromaticity diagram. These ellipses give g_{ik} metrics in the xyY space. To obtain ellipses in the u^*, v^* diagram, this metric should be transformed into the CIELUV space through XYZ space. A similar technique is applied to plot the computed ellipses of above formulas in u^*, v^* diagram.

In this paper, a comparison is done between the computed ellipses of each formula and the ellipses obtained from the BFD-P data set in the u^*, v^* chromaticity diagram. The comparison method is shown by Pant et. al. [20]. This method gives a matching ratio, $R \in (0, 1]$, which takes account of variations in the size, the shape and the orientation simultaneously for a pair of ellipses. Statistical tests are also performed between R values of two formulas. Box plot and cumulative distribution function (CDF) of two R values are also plotted. Such plots illustrate the performance of two formulas for measuring visual differences.

2. COLOUR SPACES AND FORMULATION OF ELLIPSE

2.1. Ellipse Formulation

In a Riemannian model of colour space, the infinitesimal distance between any two points is expressed by a differential quadratic form. If we consider the xy chromaticity diagram, such distance is expressed as:

$$ds^2 = g_{11}dx^2 + 2g_{12}dxdy + g_{22}dy^2 \tag{1}$$

where ds is the infinitesimal distance between two points, dx is the difference of x coordinates, dy is the difference of y coordinates and g_{11}, g_{12} and g_{22} are the coefficients of the metric tensor g_{ik}. The ellipse parameters in terms of the coefficents of g_{ik} are expressed as [12]:

$$\begin{aligned} g_{11} &= \frac{1}{a^2}\cos^2\theta + \frac{1}{b^2}\sin^2\theta, \\ g_{12} &= \cos\theta\sin\theta\,(\frac{1}{a^2} - \frac{1}{b^2}), \\ g_{22} &= \frac{1}{a^2}\sin^2\theta + \frac{1}{b^2}\cos^2\theta. \end{aligned} \tag{2}$$

The angle formed by the major axis with the positive x-axis is given by

$$\tan(2\theta) = \frac{2g_{12}}{(g_{11} - g_{22})}. \tag{3}$$

Here $\theta \leq 90°$ when $g_{12} \leq 0$, and otherwise $\theta \geq 90°$. The ellipse parameters can also be derived from the g_{ik} metric tensor by solving the above equations for a, b and θ.

2.2. CIELUV space

The CIELUV in terms of CIE(X,Y,Z) is defined as:

$$\begin{aligned} L^* &= 116\left(\frac{Y}{Y_r}\right)^{\frac{1}{3}} - 16 \\ u^* &= 13L^*\left[\left(\frac{4X}{X+15Y+3Z}\right) - \left(\frac{4X_r}{X_r+15Y_r+3Z_r}\right)\right] \\ v^* &= 13L^*\left[\left(\frac{9Y}{X+15Y+3Z}\right) - \left(\frac{9Y_r}{X_r+15Y_r+3Z_r}\right)\right] \end{aligned} \tag{4}$$

The plot of u^* and v^* values gives us UCS diagram in the CIELUV colour space. Now, to plot xyY space based BFD-P data in u^*v^* diagram, it is necsessary to transform the data from the xyY space to the CIELUV space. This is done by applying Jacobian transformation where the variables of two color spaces are related by continuous partial derivatives. This process takes following steps: First, compute the g_{ik} metric of the data according to Equation (2). Secondly, transform this metric into the XYZ color space and then to the CIELUV space. Thirdly, compute ellipse parameter from the transformed matric tensor and plot it in u^*v^* diagram. In equation form, we write:

$$g_{\Delta E^*_{uv}} = \frac{\partial(X,Y,Z)}{\partial(L,u^*,v^*)}^T \frac{\partial(x,y,Y)}{\partial(X,Y,Z)}^T g_{ik} \frac{\partial(x,y,Y)}{\partial(X,Y,Z)} \frac{\partial(X,Y,Z)}{\partial(L,u^*,v^*)}. \tag{5}$$

where $\partial(x,y,Y)/\partial(X,Y,Z)$ and $\partial(X,Y,Z)/\partial(L,u^*,v^*)$ are the Jacobian metrics. They are expressed as:

$$\begin{bmatrix} dx \\ dy \\ dY \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \\ \frac{\partial Y}{\partial X} & \frac{\partial Y}{\partial Y} & \frac{\partial Y}{\partial Z} \end{bmatrix} \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial L^*} & \frac{\partial X}{\partial u^*} & \frac{\partial X}{\partial v^*} \\ \frac{\partial Y}{\partial L^*} & \frac{\partial Y}{\partial u^*} & \frac{\partial Y}{\partial v^*} \\ \frac{\partial Z}{\partial L^*} & \frac{\partial Z}{\partial u^*} & \frac{\partial Z}{\partial v^*} \end{bmatrix} \begin{bmatrix} dL^* \\ du^* \\ dv^* \end{bmatrix} \tag{7}$$

2.3. CIEDE2000 space

The CIEDE2000 formula based on CIELAB space is expressed as a non-Euclidean metric as follows :

$$\begin{aligned} \Delta E_{00} = \Bigg[&\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 \\ &+ R_T\left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)\Bigg]^{0.5} \end{aligned} \tag{8}$$

where, lightness (S_L), chroma (S_C) and hue (S_H) are specific weighting functions. Similarly, (k_L, k_C, k_H), and R_T describe parametric factors for experimental condition and the rotation term respectively. The color coordinates used in the formula are defined from the CIELAB coordinates in the following way:

$$L' = L^*, a' = a^*(1+G), b' = b^*, \tag{9}$$

$$C' = \sqrt{a'^2 + b'^2}, h' = \arctan\frac{b'}{a'} \tag{10}$$

$$G = \frac{1}{2}\left(1 - \sqrt{\frac{C^{*7}_{ab}}{C^{*7}_{ab} + 25^7}}\right), \tag{11}$$

For formulating Remiannian metric of ΔE_{00}, we need to to use $L'C'h'$ coordinates instead of $L'C'H'$ because the coordinate H' does not exist. The metric using $L'C'h'$ coordinates gives us an approximation of ΔE_{00}. Considering $dH' = C'dh'$ [21], the approximate CIEDE2000 color difference metric tensor in the CIELUV space is

$$g_{\Delta E_{00}} = \frac{\partial(X,Y,Z)}{\partial(L^*,u^*,v*)}^T \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)}^T \frac{\partial(L',C',h')}{\partial(L,a^*,b^*)}^T \times$$
$$\begin{bmatrix} (k_L S_L)^{-2} & 0 & 0 \\ 0 & (k_C S_C)^{-2} & \frac{1}{2}C' R_T(k_C S_C k_H S_H)^{-1} \\ 0 & \frac{1}{2}C' R_T(k_C S_C k_H S_H)^{-1} & C'^2(k_H S_H)^{-2} \end{bmatrix}$$
$$\times \frac{\partial(L',C',h')}{\partial(L,a^*,b^*)} \frac{\partial(L,a^*,b^*)}{\partial(X,Y,Z)} \frac{\partial(X,Y,Z)}{\partial(L,u^*,v^*)} \tag{12}$$

where, $\partial(L',C',h')/\partial(L,a^*,b^*)$, $\partial(L,a^*,b^*)/\partial(X,Y,Z)$ are the Jacobian metrices. The derivations of these metrices can be found in the paper Pant and Farup [9].

2.4. OSA-UCS based ΔE_E space

The ΔE_E color difference formula based in the log compressed OSA-UCS color space are derived from the original OSA-UCS formula. Their definitions are expressed as follows:

$$L_E = \left(\frac{1}{b_L}\right)\ln\left[1 + \frac{b_L}{a_L}(10L_{OSA})\right], \tag{13}$$

$$C_E = \left(\frac{1}{b_c}\right)\ln\left[1 + \frac{b_c}{a_c}(10C_{OSA})\right], \tag{14}$$

$$C_{OSA} = \sqrt{G^2 + J^2}, h = \arctan(\frac{-J}{G}), \tag{15}$$

$$G_E = -C_E\cos(h), J_E = C_E\sin(h) \tag{16}$$

Expressing G_E and J_E in terms of C_{OSA}, we have:

$$G_E = -\frac{C_E G}{C_{OSA}}, J_E = -\frac{C_E J}{C_{OSA}} \tag{17}$$

The OSA-UCS color space is also related to the CIEXYZ color space. The details can be found in the paper of Huertas et al. [22]. For calculating the line element at a given point, the log-compressed OSA-UCS formula is written as:

$$(dE_E)^2 = \begin{bmatrix} dL_E & dG_E & dJ_E \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dL_E \\ dG_E \\ dJ_E \end{bmatrix}. \tag{18}$$

Then, in the similar way as mentioned for the CIEDE00, it is transformed to OSA-UCS color space and finally into the CIELUV space. From the transformed metric tensor, we compute JND ellipses in u^*, v^* chromaticity diagram. The computation of ΔE_E metric tensor is published in the previous article [19].

2.5. Ellipse pair matching ratio

The matching ratio to compare a pair of ellipses A and B is expressed as:

$$R = \frac{\text{Area}(A \bigcap B)}{\text{Area}(A \bigcup B)} \tag{19}$$

High value of R gives strong evidence that the two ellipses are closely matched and vice versa.

3. RESULTS

The ellipses of BFD-P, the ΔE_E and the ΔE_{00} in the uniform u^*, v^* chromaticity diagram are shown in Figures 1(a), 1(b), and 1(c) respectively. All the ellipses are computed at constant lightness value ($L^* = 50$). The colour centers for the computed ellipses are taken from the BFD-P data. Around the neutral region the computed and BFD-P ellipses look similar. In the blue region, ellipses of both formulas appear smaller in size than the observed ellipses. Similarly, in comparison to BFD-P, ΔE_{00} ellipses are rotated more in blue and red regions than the ΔE_E. So, the ΔE_E performs better for measuring experimental data in these region. Likewise, ellipses of both formulas are following close pattern to BFD-P in the green part.

Figures 2(a) and 2(b) are the histograms of R values of the ΔE_{00} and the ΔE_E with respect to BFD-P ellipses. The maximum R value of both formulas is approximately .92 where as the lowest value for the ΔE_{00} and the ΔE_E is .25 and .2 respectively. Ellipse pairs having maximum R values are located around neutral colour region. Similarly, in the ΔE_{00} case lowest R are found around blue and red regions while for the ΔE_E it is located in blue-violet region.

Figure 3(a) shows box plot of the R values. In such plots, the median value is marked by the central horizontal lines. The notch indicate the confidence interval of the median, and the box is bounded by the upper and lower quartiles of the grouped data. We can see that both ΔE_{00} and the ΔE_E have similar median values with slight differences. The authors also plotted cumulative distribution function (CDF) of two R values. This function describes the distribution of R values by taking the sum of each value of R having finte mean and variance. The distribution curves of two sets of R is shown in figure 3(b). They follow the similar continous normal distribution and the maximum difference between two curves is .1. Similarly, the pairwise statistical sign test of R values shows that there is no significant difference between ΔE_{00} and ΔE_E formulas.

4. CONCLUSION

In this paper, the UCS diagram based on CIELUV colour space is used to evaluate two colour difference formulas ΔE_{00} and ΔE_E for measuring the visual data. On the basis of our analysis, the authors can say that statistically, there is no significant difference between the Euclidean ΔE_E and the non-Euclidean CIEDE2000 formulas compared to the BFD-P data set. However, the performance of the CIEDE2000 is found weaker in blue and red region compared to the ΔE_E. It is interesting to note that simple Euclidean ΔE_E metric is not inferior to the complex non-Euclidean mathematical framework of ΔE_{00} for evaluating color differences.

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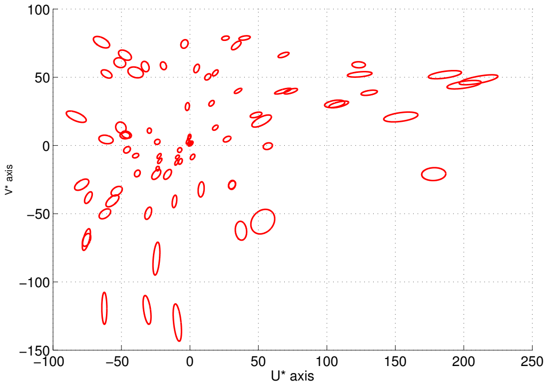
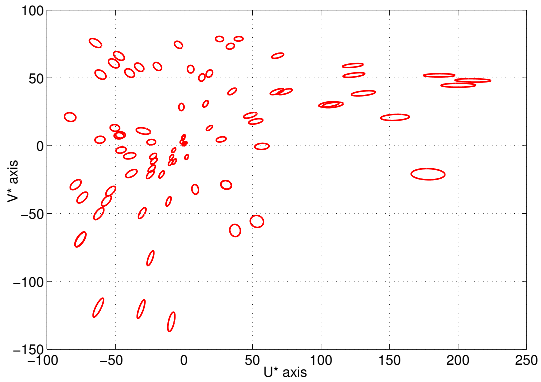
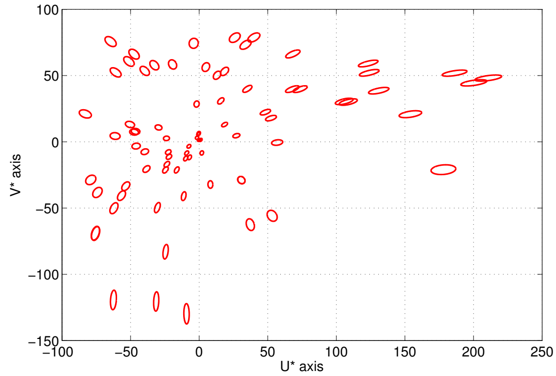


(a) BFD-P ellipses in UCS diagram.

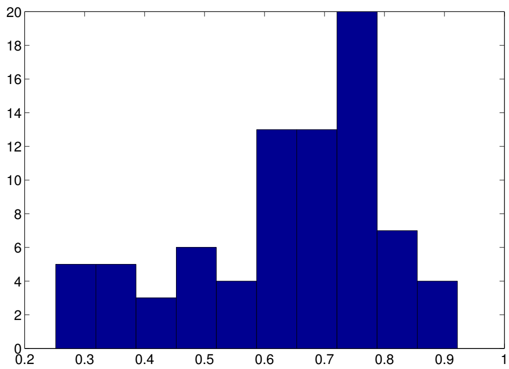


(b) CIEDE00 ellipses in UCS diagram.

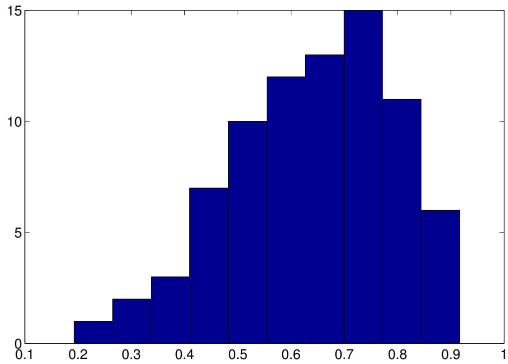


(c) OSA-UCS ΔE_E ellipses in UCS diagram .

Fig. 1. BFD-P and computed OSA-UCS ΔE_E and approximate CIEDE00 ellipses in the u^*v^* chromaticity diagram.

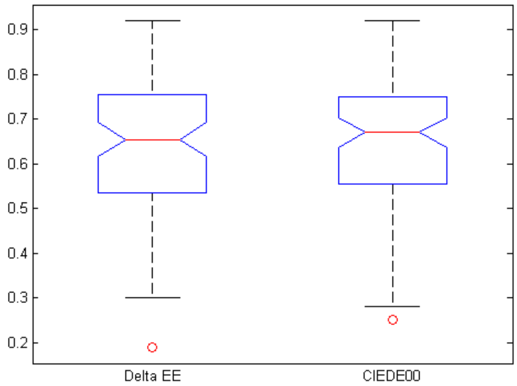


(a) CIEDE00.

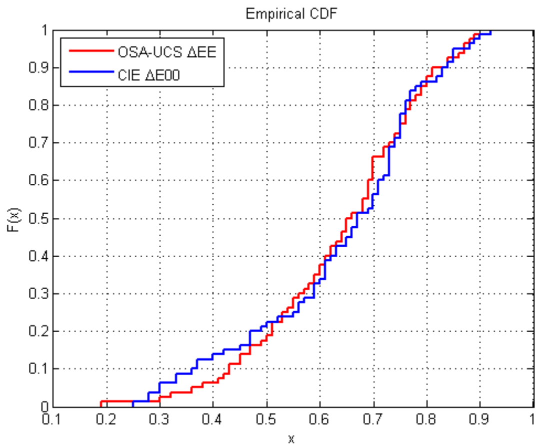


(b) OSA-UCS ΔE_E.

Fig. 2. Histogram of R values of approximate CIEDE00 and OSA-UCS ΔE_E .



(a) Box plot.



(b) CDF plot.

Fig. 3. Box and CDF plots of R values of approximate CIEDE00 and OSA-UCS ΔE_E .