

Evaluating color deficiency simulation and daltonization methods through visual search and sample-to-match: SaMSEM and ViSDEM

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ABSTRACT

Color deficient people might be confronted with minor difficulties when navigating through daily life, for example when reading websites or media, navigating with maps, retrieving information from public transport schedules and others. Color deficiency simulation and daltonization methods have been proposed to better understand problems of color deficient individuals and to improve color displays for their use. However, it remains unclear whether these color “prosthetic” methods really work and how well they improve the performance of color-deficient individuals. We introduce here two methods to evaluate color deficiency simulation and daltonization methods based on behavioral experiments that are widely used in the field of psychology. Firstly, we propose a Sample-to-Match Simulation Evaluation Method (SaMSEM); secondly, we propose a Visual Search Daltonization Evaluation Method (ViSDEM). Both methods can be used to validate and allow the generalization of the simulation and daltonization methods related to color deficiency. We showed that both the response times (RT) and the accuracy of SaMSEM can be used as an indicator of the success of color deficiency simulation methods and that performance in the ViSDEM can be used as an indicator for the efficacy of color deficiency daltonization methods. In future work, we will include comparison and analysis of different color deficiency simulation and daltonization methods with the help of SaMSEM and ViSDEM.

Keywords: sample-to-match, visual search, daltonization, color deficiency simulation, color deficiency, color image quality, image enhancement, behavioral experiment, psychology

1. INTRODUCTION

Color vision in humans is based on photoreceptors in the retina called *cones* that are differentially sensitive to wavelengths by pigments called opsins that filter different ranges of the visible electromagnetic spectrum; typically in the short, medium, and long range of the light spectrum through the S-cones, M-cones and L-cones respectively.¹ Ganglion cells in the latter parts of the human eye, i.e. middle part of the human visual system (HVS), combine the signals from the cones into pathways that roughly correspond to different perceptual attributes: A pathway for intensity, a pathway for red–green opponency, and a pathway for blue–yellow opponency.² Since all the colors that we can perceive are generated on the basis of signals from the three types of cones, this type of color vision is called *trichromatic vision*. Trichromatic vision is the norm for the majority of humans, but not for all species and for mammals it has only evolved in some primates.^{3,4} In anomalous trichromats and dichromats, the sensitivity of at least one photopigment is shifted or the photopigment may be missing all together.

Many studies indicate that color vision in general provides an evolutionary advantage reflected in behavioral advantages when differentiating between edible or non-edible fruits and foliage.⁵ Moreover, there are some indications that color assists attentional mechanisms,⁶ object recognition⁵ and possibly emotional states’ detection.⁷

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Most color deficient people do not report serious limitations in daily life and the practical consequences of color deficiency have been little researched. However, researchers agree that color deficient people have an objectively reduced capacity to differentiating colors, since certain colors are “confused” with others.⁸ Since cone sensitivity is weakened or absent in comparison to normal sighted observers, the signals reaching the ganglion cells for the red–green opponency pathway can be severely reduced, resulting in a significantly weakened response along this pathway. Since color contrast is significantly reduced, the image quality is disrupted in both anomalous trichromats and dichromats, because color edges become less visible, and it may be more difficult to retrieve crucial information from an image. In these cases, changes in colors might only be distinguished along the yellow–blue contrast, and/or lightness contrast.

For normal sighted people, in order to better understand difficulties of such color deficient observers, simulation methods may be needed. Thus, color deficiency simulation methods have been proposed to simulate the vision of color deficient observers for normal sighted observers. Most simulation methods are optimized for dichromatic color deficiency,^{9,10} whereas other can simulate anomalous trichromacy to some degree as well.¹¹ Moreover, color-deficient individuals’ processing of visual information can be improved by image enhancement methods specifically developed for color deficient observers. The so-called daltonization methods have been proposed to improve image quality for color deficient observers by increasing and/or reintroducing lost or decreased color contrast in order to regain lost information.^{11–14} Most of these methods are in turn based on color deficiency simulation methods in order to determine which colors in the image are of difficulty for color deficient observers.

Color deficiency simulation and daltonization methods still leave unresolved two main questions: 1. If and how well do simulation methods actually visualize color-deficient vision? Brettel *et al.*⁹ reported that they tested their method on only one protan and one deutan dichromat observers, who indeed could not see any differences between the original and the simulation. However, a broad survey conducted on several color deficient observers has not yet been thoroughly performed. 2. How well do daltonization methods work, i.e. do they really improve color images for color deficient observers? Some researchers have put to test their daltonization method on the so-called Ishihara plates,¹⁵ i.e. checking if color-deficient people were able to extract the correct information after daltonization. However, one should note that Ishihara plates do not correspond to natural images since the image dots do not overlap and are surrounded by a white background. However, this limitation might be avoided by updated Ishihara plates as proposed by Rizzi *et al.*,¹⁶ in which color dots actually overlap.

In this paper, we present two experiment methods allowing the analysis and comparison of multiple color deficiency simulation and daltonization methods, using sample-to-match for simulation (**SaMSEM**) and visual search for daltonization (**ViSDEM**), based on the experiments of Bramão *et al.*⁵ and by Treisman *et al.*⁶ respectively. The present paper is divided into (i) an account of the general methodology, (ii) an analysis of the current implementation, (iii) a presentation of the results with a discussion, and (iv) a final conclusion. The purpose of this paper is to test the evaluation methods, not yet to compare the simulation and daltonization methods. In other words, we want to show how our evaluation methods can be used to show that color deficiency simulation and daltonization methods really work. In a previous study from 2013,¹⁷ we presented a task-based accessibility measurement of daltonization algorithms for information graphics, namely for public transportation maps, using response time (RT) data that we tested on normal sighted observers using both color deficiency simulation and daltonization methods. The new methods follow a more standardized protocol and involve color deficient observers as well.

2. METHODOLOGY

2.1 Sample-to-match Simulation Evaluation Method (SaMSEM)

The purpose of the sample-to-match simulation evaluation method (**SaMSEM**) is to show that color deficiency simulation methods do simulate color deficiency vision adequately. We founded the method on the premise that color deficient observers will have difficulties seeing a difference between the original image and its simulation. In the three-step setup for the color deficiency simulation verification experiment (cf. Figure 1), (i) observers were firstly presented with the target image centered on the screen in front of a gray background, which was either the original or one of the simulated version of the image. (ii) The target then disappeared, and the observers looked

at a gray screen with a fixation cross in the center. (iii) Thirdly, the observers were presented with the original and one simulated version next to each other in front of a gray background, of which one image was the target version that he/she had just seen before. The observer was asked to press either the left or right arrow key on the keyboard according to whether the target image was now located on the right or the left. We recorded both whether or not the observer answered correctly, i.e. the accuracy, and how long it took to respond to the task, i.e. the response time (RT).

We hypothesized that color deficient observers would have problems spotting the difference between original and simulated versions, i.e. they would react slower, resulting in higher RTs, and make more mistakes, resulting in lower accuracy, when detecting the correct image. On the other hand, normal sighted observers should have little problems spotting the difference between the original and its simulation, resulting in faster RTs and higher accuracy. We analyzed the results by plotting the box plots of the RTs including the 95% confidence interval of the average RT among all observer groups, i.e. normal sighted, deutan and protan color deficient observers. And we plotted the accuracy with 95% confidence interval for all observer groups. We defined RT and accuracy of one group to be significantly different from another group, if their confidence intervals did not overlap.

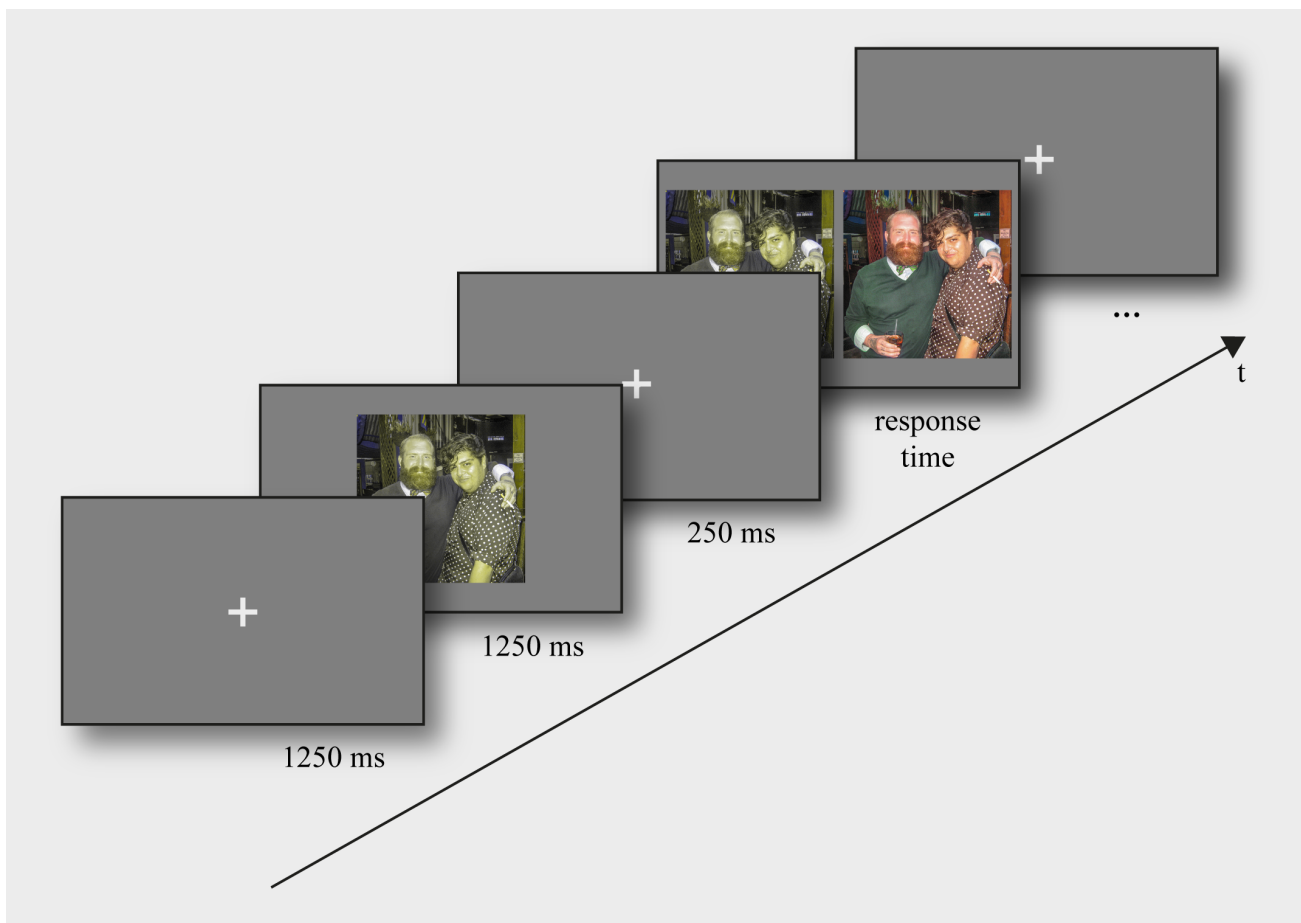


Figure 1: SaMSEM workflow: The observer is firstly presented with the target stimulus for 1250 ms before it disappears. He/she sees secondly the same motive in two different versions, both the original and the simulated version of the motive, and he/she is asked to press either the right or the left arrow key on the keyboard according to whether the target is now located on the left or on the right. The program records both the response time (RT), in ms, and the correctness, as boolean, of the subject's response.

2.2 Visual Search Daltonization Evaluation Method (ViSDEM)

The purpose of the visual search daltonization evaluation method (ViSDEM) is to show that color deficiency daltonization methods really improve color image quality for color deficient observers. We have founded the method on the assumption that color deficient observers can be enabled to retrieve relevant information quicker from the image than they could be before the image has been “daltonized”. As the previous experimentation, the color deficiency daltonization evaluation experiment consists of three steps (cf. Figure 2). (i) The observer is firstly presented a statement addressing the colors of specific objects in the image like for example “The jerseys of the wrestlers have the same color hue” or “The feathers of the bird have a different color hue than the leaves in the background.” (ii) Secondly, the observers are presented with a gray screen with a cross for fixation in the center of the screen. (iii) Thirdly, they see the target image in the center, the previously shown statement below the image. The observer is asked to press the left arrow key of the keyboard, labeled with “AGREE”, if he/she agrees that the statement is correct for the presented image, respectively press the right arrow key of the keyboard, labeled with “DISAGREE”, if he/she disagrees that the statement is correct for the image. We recorded whether or not the observer answers correctly, i.e. the accuracy, and how long he/she takes to respond to the task, i.e. the RT.

For each set of questions we chose different motives like wrestler jerseys, fruits in front of foliage, etc. For each motive, we changed the colors in order to create different variants of the same motive such that the colors would either become (i) difficult for nobody (DN), (ii) difficult for color deficient observers (DC), or (iii) difficult for everybody (DE). We hypothesized that color deficient observers would have problems retrieving information from both the DC and DE variants of the motives resulting in slower RTs and lower accuracy as compared to the DN variant, whereas normal sighted people would not have problems for the DC variants of the motives resulting in no significant different accuracy and/or RTs. We analyzed the results by plotting the box plots of the RTs including the 95% confidence interval of the average RT among all image variants, i.e. DN, DC and DE. And we plotted the accuracy with 95% confidence interval for all image variants.

3. IMPLEMENTATION

We setup the experiment on two computers in a closed room with D50 fluorescent lighting dimmed to 30 lux. For the calibration of the computer display, an i1Pro spectrophotometer and i1Match Software was used to determine the display white point of 6500k, gamma of 2.2 and the brightness of $120 \frac{cd}{m^2}$. All images were assigned with sRGB profiles. For showing images, we used the PsychoPy2 library, which measured RTs and accuracies. of the analysis of the data was done with Python libraries, namely Pandas for organization and retrieving of the data, and Numpy, SciPy and Matplotlib for the analysis of the data.

For SaMSEM, we tested four different simulation methods on 44 different motives: The motives were chosen according to different attributes like predominant color hue/s, protan/deutan/normal color contrast, overall lightness and saturation, skin, sky and grass memory colors, large areas of the same color, color transitions, fine details and busyness based on the classification proposed by Pedersen.^{18,19} The simulation algorithms were the ones proposed by Viénot *et al.*,¹⁰ an improved method of the Viénot algorithm, an algorithm presented by Brettel *et al.*,⁹ and an algorithm presented by Kotera.¹¹ In addition to these, we had a reference dummy simulation method that converted the color images into black-and-white images. For all these methods, we computed the simulated versions for protanopia and deuteranopia, since we were able to recruit only color-deficient observers belonging to either one of these categories.

For ViSDEM, we tested the method on 7 different image sets: The motives have been chosen according to their color contrast with focus on red-green opponency, and preferably images have been chosen that could be associated to a specific task. The chosen task was whether certain elements in the image have different color hue (for group A), or if these elements have same color hue (for group B). Groups were assigned randomly in the beginning of the experiment. The daltonization algorithms were the ones proposed by Anagnostopoulos *et al.*,¹² Kotera,¹¹ Kuhn *et al.*,¹³ and Huang *et al.*¹⁴ As before, we used a reference dummy daltonization method that convert color images into black-and-white images. For all methods, we computed the daltonized versions for protanopia and deuteranopia. In this paper, however, we do not consider the daltonized version, but only the DN, DC and DE variants of the original. Again, we hypothesize that the three variants will lead to no significant

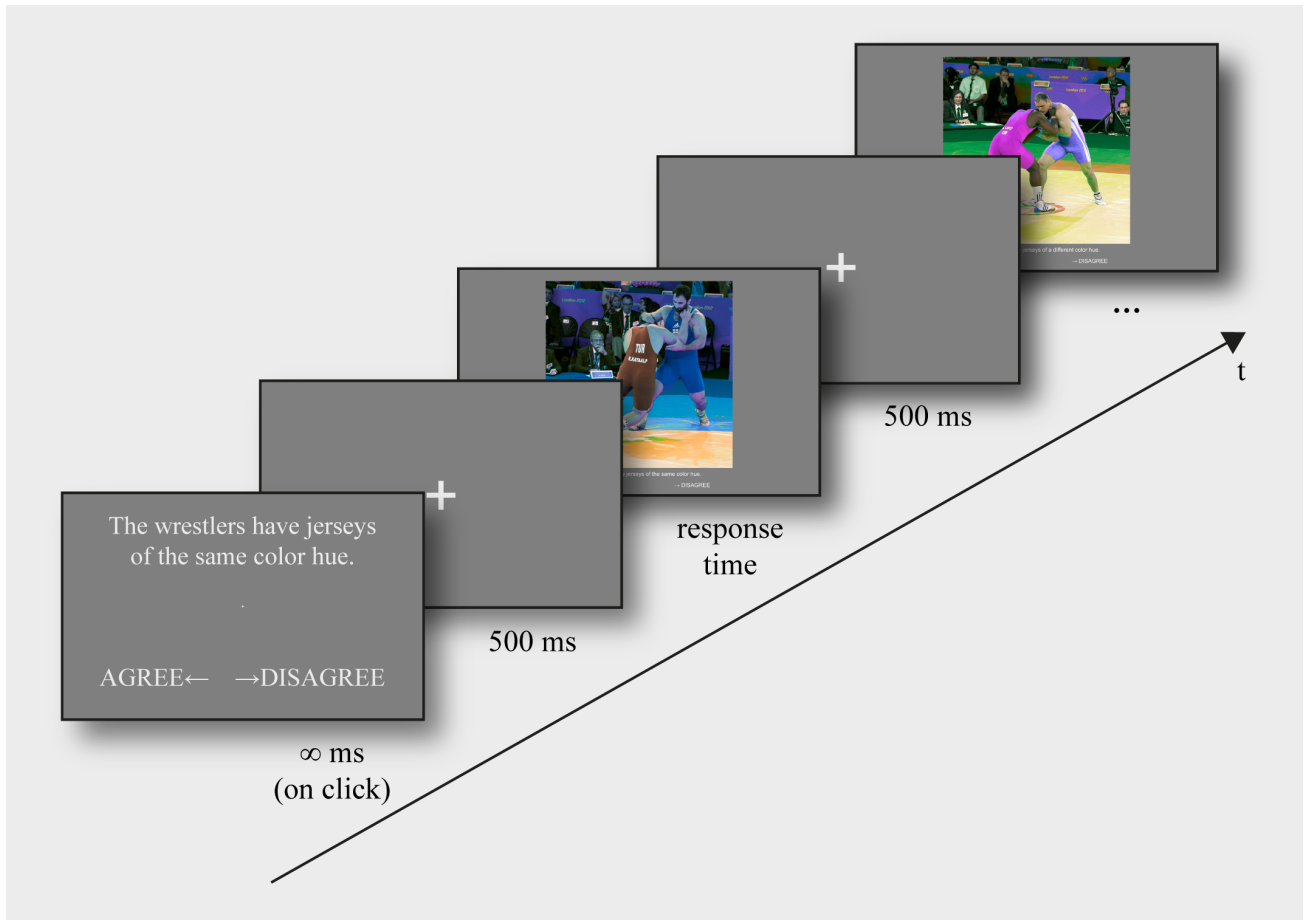


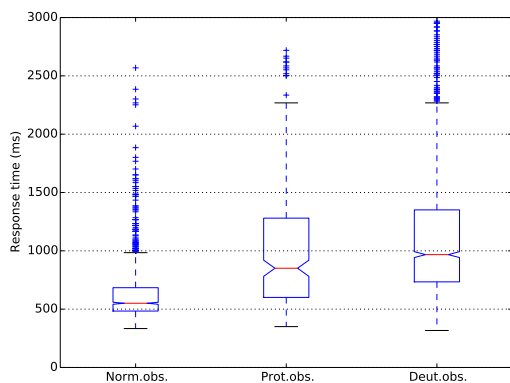
Figure 2: ViSDEM workflow: The observer is firstly presented with the a statement that addresses some of the information content in the images. The images differ from set to set, and each set may contain different motives. He/she sees secondly one of the motives in the set in different variants. There are variants containing colors that are difficult for nobody (DN), difficult for color deficient people (DC), or difficult for everybody (DE). He/she is then asked to press either the left arrow key of the keyboard if she agrees, or the right arrow key of the keyboard if he/she disagrees with the statement. The program records both the response time (RT), in ms, and the correctness, as boolean, of the subject's response.

different RTs and accuracies for normal sighted observers. However, we assume that the RTs for DC variants will be significantly higher and the accuracy will be significantly lower for color deficient observers than compared to the other two variants and than compared to normal sighted observers since we chose colors for the DC variants that would be difficult for color deficient observers.

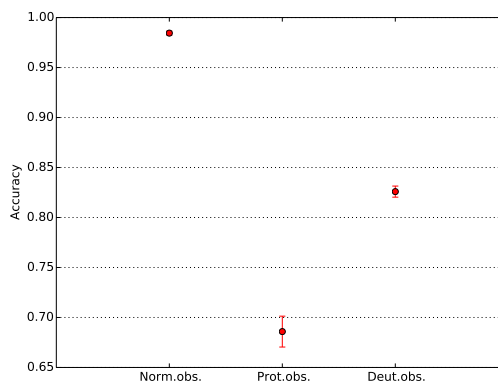
For SaMSEM, we tested our implementation on 24 observers, 10 of which were normal sighted, 12 of which were deutan color deficient, and 2 of which were protan color deficient. For ViSDEM, we tested our implementation on 23 observers, 10 of which were normal sighted, and 13 of which were deutan color deficient. All observers were aged 20 to 65, normal or otherwise corrected to normal vision. We tested color deficiency using four different kind of color vision tests, namely (i) the Ishihara test plates,¹⁵ (ii) the HRR color vision test,²⁰ (iii) the Farnsworth D15 Hue test,²¹ (iv) and the Lanthony D15 Desaturated Hue test.²² Since none of the tests are accurate enough to separate dichromats from anomalous trichromats – for this we would have to use an anomaloscope –, we summarized both deuteranopes and deuteranomalous in a group referred to as deutan color deficient observers, and both protanopes and protanomalous in a group referred to as protan color deficient observers.

4. RESULTS

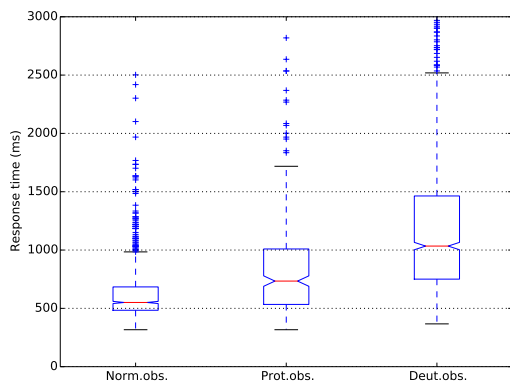
For SaMSEM, we combined the data from all images collapsed on all observers after grouping them into three groups of “normal sighted”, “protan color deficient” and “deutan color deficient”. We then plotted the RTs (cf. Figures 3a and 3c) and accuracies (cf. Figures 3b and 3d). The box in each box plot represents the upper and lower quantile respectively, whereas the whiskers represent the range of the distribution. The red bar inside of the box represents the average RT whereas the notch of the box represents the 95% confidence interval of the average RT. The confidence interval of the notches is computed with a Gaussian-based asymptotic approximation²³ through Python’s Matplotlib library. The dot in the accuracy graph represents the average accuracy within that particular group, and the handle bar represents its 95% confidence interval. The confidence interval of the accuracy has been computed by $CI = p \pm 1.96 \cdot se$, where $se = \sqrt{\frac{p \cdot (1-p)}{N}}$, where N is the number of observations.



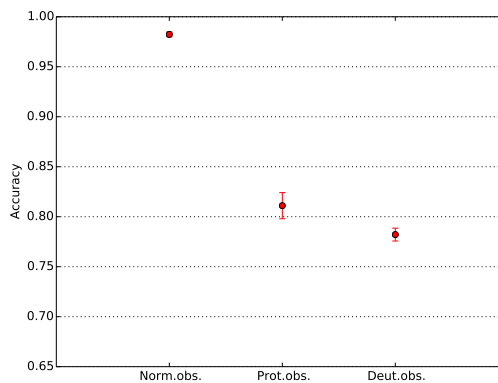
(a) RTs for SaMSEM simulating protanopia



(b) Accuracies for SaMSEM simulating protanopia



(c) RTs for SaMSEM simulating deuteranopia

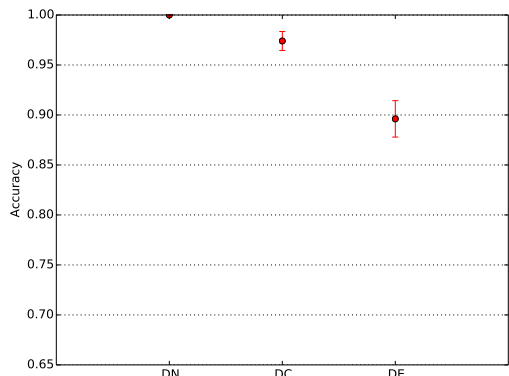
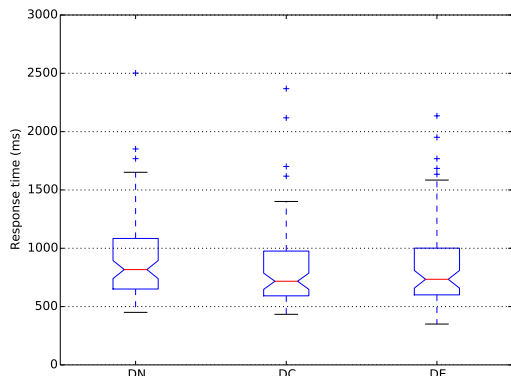


(d) Accuracies for SaMSEM simulating deuteranopia

Figure 3: Comparing the results from SaMSEM of normal sighted, protan and deutan color deficient observers. The upper row depicts the results from protanopia simulation methods, whereas the lower row depicts results from the deuteranopia simulation methods. It can be seen that color deficient observers have a significantly higher RT than normal sighted observers, and that normal sighted observers have a significantly higher accuracy than color deficient observers. This is in accordance with our hypothesis. More explanations in the text (cf. Section 4).

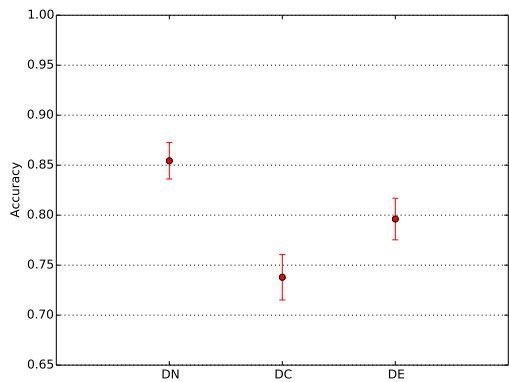
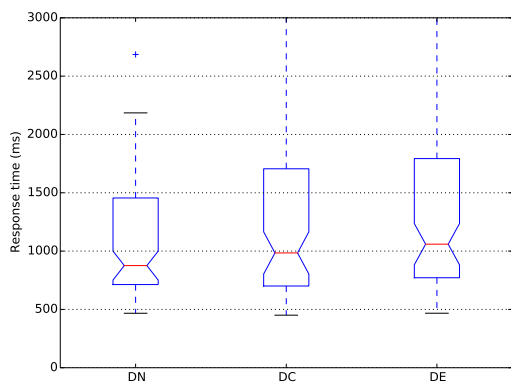
For ViSDEM, we combined the data from all motives of all sets collapsed of normal sighted and deutan color deficient observers. For the motive variants *difficult for nobody* (DN), *difficult for color deficient observers* (DC), and *difficult for everybody* (DE), we then plotted the RTs (cf. Figures 4a and 4c) and accuracies (cf. Figures 4b and 4d). The box of the box plots represents the lower and upper percentile, whereas the whiskers contain the

range of the distribution. The red bar inside of the box represents the average RT whereas the notch of the box represents the 95% confidence interval of the average RT. The dot in the accuracy graph represents the average accuracy within that particular group, and the handle bar represents the 95% confidence interval. Calculations have been done as explained before for SaMSEM.



(a) RTs for ViSDEM of normal sighted observers

(b) Accuracies for ViSDEM of normal sighted observers



(c) RTs for ViSDEM of deutan color deficient observers

(d) Accuracies for ViSDEM of deutan color deficient observers

Figure 4: Comparing the results from ViSDEM of normal sighted and deutan color deficient observers. The upper row depict the results of normal sighted and the lower row depict results from deutan color deficient observers. It can be seen that the accuracy of deutan observers is significantly lower for the DC variant than for any other variant, and also significantly lower than compared to the normal sighted observers. It is interesting to note that for both normal sighted and deutan color deficient observers the accuracy of DE is significantly lower than the accuracy of DN. The average RTs for all three variants of normal sighted observers seem to be more or less the same. The average RT of the DC variant for deutan observers seems to have a tendency to be higher than the DN variant however, there is no statistically significance observable in the current experimentation data.

5. DISCUSSION

For SaMSEM, the average RT for normal sighted observers was significantly lower than for both deutan and protan color deficient observers, given that the confidence intervals did not overlap. Typically, significance is obtained when the mean of one group is outside the confidence interval of another group. Also, the accuracy of normally-sighted observers was significantly higher than that of both deutan and protan color deficient observers.

Both results indicate that color-deficient people have indeed difficulties separating the original from the simulated versions since the confidence intervals do not overlap. Thus, the simulations do indeed illustrate the color deficiencies adequately. Moreover, the accuracy of protan color deficient observers was significantly lower than that of deutan color-deficient observers for the protanopia simulations. Accuracy of deutan color deficient observers was significantly lower than the accuracy for protan color-deficient observers for the deuteranopia simulations. The RTs of deutan color deficient observers were significantly higher for both deuteranopia and protanopia simulations. Since we could only test two protan color deficient observers, the present results for protan color deficient observers should be taken with caution.

For ViSDEM, there were no significant variations in RTs of normal sighted observers when comparing the DN, DE and DC variants of the images. Also, the accuracy for DN and DC of normal sighted observers were not significantly different. Both observations indicate that, for normal sighted observers, in both image variants, i.e. DN and DC, the confusing colors look sufficiently different. Moreover, the accuracy of the DE variant was significantly different from both the DN and DE variants. On the other hand, there was a tendency towards a difference between RTs and performance of deutan color deficient observers for DC variants as well as for DN and DE variants. That is, the DC has a significantly lower RT, and a significantly lower accuracy. Accuracy indicated that color deutan color deficient observers did indeed have more difficulties interpreting the correct colors in the DC image. Moreover, the accuracy of the DE variants was significantly lower than the accuracy of the DN variant, as before for normal sighted observers. To summarize, we can conclude that for both groups, normal sighted and deutan color deficient observers, images that have been chosen to be “difficult for nobody” do indeed result in the highest accuracy within each group, whereas the image that have been chosen to be “difficult for nobody” do indeed result in significantly lower accuracy than the DN variant images. The images that have been chosen to be “difficult for color deficient people” do indeed result in the lowest accuracy for deutan color deficient observers, but result in comparable accuracies as the DN variant images for normal sighted observers. Thus, the accuracy results do support our hypothesis. RTs on the other hand did not give any statistically sufficient answers to our hypothesis.

In future work, we will compare different simulation and daltonization methods, in order to analyze if and how one method performs better than another. More precisely, (i) if alternative simulation methods show significant differences in efficiency of response or accuracy we assume that such a simulation method performs better than another. This method would result in slower RTs and lower accuracies. Also, (ii) if different daltonization methods show significant differences in RTs and accuracy we assume that one simulation method performs better than another.

6. CONCLUSION

We introduced two methods to evaluate color deficiency simulation and daltonization methods through sample-to-match (SaMSEM) and visual search (ViSDEM) tasks. We showed that both RTs and accuracy can be used as indicators of whether a color deficiency simulation works, and that accuracy can be used as an indicator of whether a color deficiency daltonization works. In future work, we will expand our research to test whether both proposed methods can be used to compare and rank different color deficiency simulation and daltonization methods.

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